Fifth International Olympiad, 1963

1963/1.

Find all real roots of the equation

$$\sqrt{x^2 - p} + 2\sqrt{x^2 - 1} = x,$$

where p is a real parameter.

1963/2.

Point A and segment BC are given. Determine the locus of points in space which are vertices of right angles with one side passing through A, and the other side intersecting the segment BC.

1963/3.

In an n-gon all of whose interior angles are equal, the lengths of consecutive sides satisfy the relation

$$a_1 \geq a_2 \geq \cdots \geq a_n.$$

Prove that $a_1 = a_2 = \cdots = a_n$.

1963/4.

Find all solutions x_1, x_2, x_3, x_4, x_5 of the system

$$\begin{aligned} x_5 + x_2 &= yx_1 \\ x_1 + x_3 &= yx_2 \\ x_2 + x_4 &= yx_3 \\ x_3 + x_5 &= yx_4 \\ x_4 + x_1 &= yx_5, \end{aligned}$$

where y is a parameter.

1963/5.

Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$.

1963/6.

Five students, A, B, C, D, E, took part in a contest. One prediction was that the contestants would finish in the order $ABCDE$. This prediction was very poor. In fact no contestant finished in the position predicted, and no two contestants predicted to finish consecutively actually did so. A second prediction had the contestants finishing in the order $DAECB$. This prediction was better. Exactly two of the contestants finished in the places predicted, and two disjoint pairs of students predicted to finish consecutively actually did so. Determine the order in which the contestants finished.