



Kuwa Gatatu, 15. Nyakanga 2026

Ikibazo 1 Hari imibare ishyitse 2026 iruta 1 yanditse ku kibaho, ntabwo biri ngombwa ko iba itandukanye. Kuri "move", Confucius ahitamo imibare ibiri ishyitse $m > 1$ na $n > 1$ ku myanya itandukanye ku kibaho akayisimbuza

$$\gcd(m, n) \quad \text{na} \quad \frac{\text{lcm}(m, n)}{\gcd(m, n)}.$$

Arakomeza agakora "moves" igihe cyose byashoboka kuyikora.

- (a) Erekanako, hatitawe ku mahitamo Confucius yakora, nyuma ya "moves" zibarika, ari umubare umwe gusa ushyitse M uzasigara ku kibaho uruta 1.
- (b) Erekanako agaciro ka M kadaterwa n'amahitamo ya Confucius.

(Menya ko $\gcd(x, y)$ bisobanuye umubare munini ushoboka ku buryo x igabanyika na wo ndetse na y igabanyika n'uwo mubare, noneho $\text{lcm}(x, y)$ bikavugaga umubare muto ushoboka ugabanyika na x ndetse ukanagabanyika na y .)

Ikibazo 2 Reka ABC ibe mpandeshatu unareke utudomo M na N tube turi muri kimwe cya kabiri k'impande AB na AC , uko byakurikiranye. Reka utudomo K na L dushyirwe mu imbere muri mpandeshatu BMC na BNC , uko byakurikiranye, ku buryo K iba irimo imbere muri mpandeshatu ABL na L ikaba irimo imbere muri mpandeshatu AKC . Fata ko

$$\angle KBA = \angle ACL, \quad \angle LBK = \angle LNC, \quad \text{na} \quad \angle LCK = \angle BMK.$$

Reka O ibe "circumcentre" ya mpandeshatu AKL . Erekanako $OM = ON$.

Ikibazo 3 Reka n ibe umubare ushyitse uruta zero. Liu Bang na Xiang Yu bafite inkoni fite uburebure bungana na 1 none bashakaga kuyigabana. Liu ashyira utudomo tutarenze n ku nkoni, noneho na Xiang agashyira utudomo tutarenze n ku nkoni. Utudomo bose bashyira ho turatandukanye (nta na kamwe wakwiriranye n'akandi). Noneho, inkoni bayikatamo uduce bakagenda bakatira ahantu hose hari akadomo. Nyuma, bagenda basimburana buri wese agafata agace katariyitirirwa n'umwe muri bo akakiyitirira (kakaba ake), aha ni Liu utangira. Intego ya buri mukinnyi muri bo ni ukugira uburebure bunini bushoboka bw'uduce twe udushyize hamwe. Kuri buri n , Vuga umubare munini ushoboka c ku buryo Liu yaba yizeye ko yabona uburebure bwose hamwe bw'uduce twe bungana byibuze na c , hatitawe ku buryo Xiang yakina.



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Ikibazo 4 Shan-Yu na Mulan bari gukina umukino. Reka θ ibe imfuruka izwi n'abo bakinnyi bose kandi $0^\circ < \theta < 180^\circ$. Ku ntangiriro, Shan-Yu akora mpandeshatu $\mathcal{T}$ ikoze mu rupapuro ifite imfuruka ashaka yihitiyemo. Noneho, mu buryo bugenda bwisubiramo bakora ibi bikurikira:

- Iyo $\mathcal{T}$ ifite byibuze imfuruka imwe ingana na θ , umukino uhita urangira Mulan akaba atsinze.
- Iyo atari uko bigenze, Mulan ahitamo akadomo P ku muzenguruko wa $\mathcal{T}$, katari imwe muri "Vertices" za $\mathcal{T}$. Ahita ahuza ako kadomo P na "Vertex" ya $\mathcal{T}$ itari ku murongo P iriho, noneho $\mathcal{T}$ akayicamo mpande eshatu ebyiri anyujije muri wa murongo wahuje P na "vertex" bitari ku murongo umwe.
- Shan-Yu ahitamo imwe muri izo mpandeshatu akayijugunya ntibongere kuyikoresha. Mpandeshatu atajugunye (isigaye) ihinduka $\mathcal{T}$ nshya bigakomeza gutyo.

Ni izihe ngano θ igomba kugira kugira ngo Mulan yizere intsinzi nyuma y'inshuro zibarika, hatitawe ku buryo bwose Shan-Yu yakina?

Ikibazo 5 Let $\mathbb{R}_{>0}$ be the set of positive real numbers. Determine all functions $f: \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$ such that

$$\sqrt{\frac{x^2 + f(y)^2}{2}} \geq \frac{f(x) + y}{2} \geq \sqrt{xf(y)}$$

for every $x, y \in \mathbb{R}_{>0}$.

Ikibazo 6 Reka $a_1, a_2, a_3, \dots$ ibe urutonde rutarangira rw'imibare ishyitse iruta 1. Fata ko kuri buri mubare ushyitse n , umubare a_{n+1} ariwo mubare muto ushyitse uruta a_n ku buryo $\gcd(a_{n+1}, a_i) > 1$ kuri buri $i = 1, 2, \dots, n$. Ereka ko habaho imibare ishyitse T na L ku buryo

$$a_{n+T} = a_n + L$$

kuri buri mubare ushyitse n . (Fata ko $\gcd(x, y)$ bivuze umubare munini ushoboka ku buryo x igabanyika na wo ndetse na y ikaba igabanyika na wo.)